The moment-gradient factor in lateral– torsional buckling of castellated steel beams under mid-span concentrated load

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ABSTRACT

increasing the value of (h_o/H) and decreasing the value of (h_w/t_w) . On the other hand, the influence of (b_4/h_w) has noticed effect on the value of C_b especially in big opening. The studies extended to investigate the non-compact sections to compare the values of C_b in the case of compact and non-compact sections with the predicted design codes. It is found that, the values of C_b in short spans have lower values than the predicted design codes. Hence, simplified empirical Equations of C_b are proposed for compact and non-compact section based on the best fit of the values of FE.

In this paper, the elastic lateral stability of castellated steel beams with hexagonal opening under concentrated load is conducted. 3D elastic finite element (FE) model is performed by using FE software package ABAQUS. A parametric study is made on compact sections to investigate the effect of the slenderness ratio of web (h_w/t_w) , the size opening (h_o/H) , spacing of opening (b_4/h_w) and slenderness ratio of flange (b_f/t_f) , on the behavior of moment-gradient factor (C_b) . The results indicate that by increasing the values of α_{LB} parameter the values of C_b decrease. In addition, the values of C_b decrease with

1 Introduction

The benefit to use beams with web opening in the building is to allow services ducts to pass. Castellated beams are varieties of girder with generally circular or rectangular or hexagonal web openings, distributed along the beams with regular intervals. The process of built-up castellated beams is cutting the web in zigzag pattern along its centerline and then rejoining the two halves by welding. This process leads into an increase in bending capacity and a reduction of the weight of constructions.

the beam allowing this type of beams to be used for medium to long spans constructions. Many researchers investigated the non-linear behaviour of castellated steel beams using experimental and analytical studies [1-5]. Zirakian and Showkati [1] tested six full-scale castellated beams for studying the distortional buckling behaviour. HOSAIN and SPEIRS [2] performed experimental program on 12 simply supported castellated steel beams to study the effect of the

geometry of opening on different modes of failure and ultimate strength of such beams. On the other hand, Showkati et al. [3] focused on the effect of exciting elastic braced systems on the behaviour castellated beams. Based on analytical study Ellobody [4] investigated a parametric study taking the effect of different cross-section geometries, beam length, steel strength and non-dimensional slenderness on the failure loads and the interaction of buckling modes in castellated steel beams. Soltani et al. [5] used nonlinear FE analysis to investigate the ultimate behavior of steel castellated beams with hexagonal and octagonal openings. Mohebkah [6] studied the moment-gradient factor in lateral-torsional buckling on inelastic castellated beams taking the effect of slenderness parameter in consideration. On the other hand, Sweedan [7] proposed an empirical formula of a moment modification factor for I-shaped cellular beams related to lateral buckling (LTB and LDB) modes only. The above literature review indicated that the most studies concentrated on the nonlinear behaviour of castellated beams and very limited studies were studied the elastic later-torsional buckling of that beams. Hence, the contributions of the current paper can be summarized as follows.

- Giving more information about the effect of key parameters related on the geometry of castellated beams with compact sections on the behavior of moment-gradient factor (C_b).
- Discussing the values of C_b in the case of compact and non-compact sections compared with the predicted design codes.
- Suggesting simplified empirical equations of C_b for compact and non-compact section based on the best fit of the values of FE.

2 Finite element modeling

2.1 Description of the model

The numerical program was conducted herein on full-size hexagonal castellated steel beams using the ABAQUS finite element program (version 6.8) [8]. Typical FE model was illustrated in Fig. 1 which defined all the dimensions of the current study beams. To study the C_b factor of these beams, three hundred eighty six finite element models were created on compact sections according to Euro code 3[9] for the two cases of loads; pure moment and mid-span concentrated loads. Another two hundred and seventy six finite element models were conducted on non-compact sections according to Euro code 3[9] for the two cases of loading. The considered cross-sections of the web for compact and non-compact sections defined by h_w/t_w values vary between (35 to 85) and (87 to 144), respectively. Also, the flange cross-section of compact and non-compact sections were defined by ($b_f/t_f = 11, 13$ and 14) and ($b_f/t_f = 13$ and 14), respectively. The hole size of openings were characterized by ($h_o/H = 0.3, 0.5$ and 0.8) and the values of b_4/h_w were 1, 1.5 and 2 for the compact and non-compact sections. The spans of the beams were varied between (3 to 10m) with increasing of (1m) to include the short, medium and long spans. The boundary conditions of the beams were simply supported at the end sections. S4R element characterizes with six degrees of freedom per node was used in the model. Mesh size of 10 mm was considered after many trials to get the accurate results. The steel material has been modelled as linear elastic material with Young's modulus $E_0 = 200\text{GPa}$ and Poison's ratio of 0.3.

2.2 Verification of the FE model

To verify the performance of the 3D FE model, a span of ($L = 3$ to 10m) with

Fig.1: Dimensions of a castellated steel beams with hexagonal opening

$b_f = 150$ mm, $t_f = 10.7$ mm, $h_w = 278.6$ mm, $t_w = 7.1$ mm, $H = 300$ mm, is used to conduct an eigenvalue analysis of different beams to identify their critical moment M_{o-cr} values that correspond to the lateral torsional buckling mode.

The verification firstly was made on solid simply supported beams subjected to uniform end moments by evaluating the critical buckling moment. The results of FE were compared with the analytical critical buckling moment M_{o-cr} given by Ref. [10] as given below. Sample of the results were summarized in Table 1.

$$M_{o-cr} = \frac{\pi}{L} \sqrt{EI_y C_1 \left(1 + \frac{C_2}{C_1} \frac{\pi^2}{L^2}\right)} \quad (1)$$

Where L is the unbraced span of the beam and I_y is the minor – axis moment of inertia of the cross- section. In Eq. (1), C_1 and C_2 are the torsional and warping rigidities, respectively; see Eq. 2 and 3.

$$C_1 = GJ \quad (2)$$

$$C_2 = EC_w \quad (3)$$

Where G is the shear modulus of elasticity, J is the torsional constant the cross section, E is Young's modulus of elasticity and C_w is the warping torsional constant of the cross- section.

$$J = \sum \frac{bt^3}{3} \quad (4)$$

$$C_w = \frac{t_f b_f^3 h^2}{24} \quad (5)$$

$$G = \frac{E}{2(1+\nu)} \quad (6)$$

$$I_x = \frac{1}{12} [h_w t_w^3 + 2t_f b_f^3] \quad (7)$$

The mean values of M_{cr-FE}/M_{o-cr} was 1.03 with stander deviation was 0.0073. Other verification was made on simply supported castellated beams subjected to uniform end moments; see Fig.1. The results of the quantitative comparison between analytical solutions and finite element predictions were summarized in Table 2 as a sample of results. The mean values of M_{cr-FE}/M_{o-cr} was 1.02 and its stander deviation was 0.011.

$$M_{o-cr} = \frac{\pi}{L} \sqrt{EI_{y(net)} C_1 \left(1 + \frac{C_2}{C_1} \frac{\pi^2}{L^2}\right)} \quad (8)$$

$$I_{x(net)} = \frac{1}{12} [(h_w - h_o) t_w^3 + 2t_f b_f^3] \quad (9)$$

$$J_{net} = \frac{2}{3} b_f t_f^3 + \frac{1}{3} (h_w t_w^3 - h_o t_w^3) \quad (10)$$

Hence, the results of verifications indicated a good agreement between the FE and the analytical method.

3 Results of FE

3.1 General

In this part, the influence of beam geometry on the moment gradient factor with the parameter α_{LB} given in Eq. 11 was discussed. Firstly, compact sections were conducted with various parameters as described in section 2.1 to investigate there effects on the moment gradient factor. Secondly, the study was directed to compare the values of C_b with the predicted codes. Then, the study was extended to

investigate the effect of non-compact sections on the values of C_b .

$$\alpha_{LB} = \frac{\pi}{L} \sqrt{\frac{C_2}{C_1}} \quad (11)$$

were given previously in C_1 and C_2 Where Eqs. 2 and 3.

3.2 Effect of (h_o/H) ratio

Figs. 2(a) and 2(b) show the relationships between C_b coefficient and α_{LB} parameter with $(b_f/t_f=14.02)$ and $(b_4/h_w=1)$. From the results it can be concluded that by increasing the values of α_{LB} the values of C_b decreases. Also, the values of C_b decrease with increasing the ratio of hole size. In addition, for the web plates with higher hole size $(h_o/H=0.8)$ it can observe a clear reduction in the value of C_b especially for high value of α_{LB} .

Figs. 3(a) and 3(b) present the relationships between C_b coefficient and α_{LB} parameter with $(b_f/t_f=14.02)$ and $(b_4/h_w=2)$. General trend was observed in the figure the increasing of the values of α_{LB} causes decreasing in the C_b values. On the other hand, by comparing between Fig. 2 and Fig. 3 it can be found that the increasing of the spacing of opening to the value of $(b_4/h_w=2)$ has no effect for the different hole size on the reduction of C_b .

3.3 Effect of (h_w/t_w) ratio

Fig. 4 shows the relationships between C_b coefficient versus α_{LB} parameter for section with $(b_f/t_f=14.02)$, $(b_4/h_w=1)$, $(h_o/H=0.3)$ and $(h_o/H=0.5)$ as a sample of results. The results indicated that the value of C_b decreases with increasing the value of α_{LB} . Also, for all hole size the value of C_b increases with increasing the value of (h_w/t_w) .

3.4 Effect of (b_4/h_w) ratio

Figs. 5 and 6 illustrate the relationships between C_b coefficient and α_{LB} parameter for $(b_f/t_f=14.02)$, $(h_o/H=0.5)$ and $(h_o/H=0.8)$, respectively. It was observed that with increasing the spacing between the opening (b_4/h_w) the value of C_b increases. This increase is clear observed in the high size opening $(h_o/H=0.8)$ than the lower $(h_o/H=0.5)$. Hence, it can be concluded that the influence of (b_4/h_w) was notice in the high size opening.

3.5 Effect of (b_f/t_f) ratio

Fig. 7 shows C_b coefficient versus α_{LB} parameter relationships for $(b_4/h_w=1)$ and $(h_o/H=0.5)$. From the fig.7 it can be seen that by decreasing the slenderness ratio of flange (b_f/t_f) the value of C_b decreases.

4. The moment gradient coefficient (C_b) for hexagonal castellated steel beam with codes.

In this section, a comparison between the results of C_b from FE for the previous compacted sections and additional non-compact sections presented in Figs. 8 and 9 were created to investigate the effect of various types of cross-sections on the values of C_b in all ranges of spans. For the compact sections it was found that the values of C_b were different from the predicted code; see figures from 2 to 7. The same observation was obtained for the non-compact sections with different values of C_b as can be seen in Figs. 8 and 9. Hence, in this section two simplified new equations were suggested for the compact sections and non-compact sections with respect α_{LB} parameter as a variable function. Eqs. 12 and 13 illustrate the proposed equations of C_b for the compact and non-compact sections, respectively. By comparing the results of C_b from the

Table 1: Comparison between finite element and theoretical buckling moments for solid beams

H [mm]	b _f [mm]	t _f [mm]	t _w [mm]	L [mm]	h _w [mm]	M _{cr-FE} [kN.m]	M _{o-cr} [kN.m]	M _{cr-FE} /M _{o-cr}
300	150	10.7	7.1	2000	278.6	475.0	469.10	1.01
300	150	10.7	7.1	3000	278.6	235.3	228.60	1.03
300	150	10.7	7.1	4000	278.6	147.8	142.93	1.03
300	150	10.7	7.1	5000	278.6	105.6	102.07	1.03
300	150	10.7	7.1	6000	278.6	81.6	78.95	1.03
300	150	10.7	7.1	7000	278.6	66.1	64.30	1.03
300	150	10.7	7.1	8000	278.6	55.8	54.27	1.03
300	150	10.7	7.1	9000	278.6	48.1	46.99	1.02
300	150	10.7	7.1	10000	278.6	42.4	41.46	1.02
Mean								1.03
Stander dev.								0.0073

Table 2: Comparison between finite element and theoretical buckling moments for castellated hexagonal beams

b _f [mm]	t _f [mm]	t _w [mm]	L [mm]	H _{Cast.} [mm]	h _{wcast} [mm]	b ₄ [mm]	$\frac{b_4}{h_{wcast}}$	M _{cr-FE} [kN.m]	M _{o-cr} [kN.m]	M _{cr-FE} /M _{o-cr}
150	10.7	7.1	2000	341.8	320.4	320.4	1	523.5	525.37	1.00
150	10.7	7.1	3000	341.8	320.4	320.4	1	254.1	251.05	1.01
150	10.7	7.1	4000	341.8	320.4	320.4	1	157.1	153.98	1.02
150	10.7	7.1	5000	341.8	320.4	320.4	1	110.7	108.15	1.02
150	10.7	7.1	6000	341.8	320.4	320.4	1	84.7	82.52	1.03
150	10.7	7.1	7000	341.8	320.4	320.4	1	68.3	66.50	1.03
150	10.7	7.1	8000	341.8	320.4	320.4	1	57.1	55.65	1.03
150	10.7	7.1	9000	341.8	320.4	320.4	1	49.1	47.86	1.03
150	10.7	7.1	10000	341.8	320.4	320.4	1	43.1	42.01	1.03
Mean										1.02
Stander dev.										0.011

Figs. 2: Variation of the C_b coefficient with the α_{LB} parameter for Castellated steel beams

Figs. 3: Variation of the C_b coefficient with the α_{LB} parameter for castellated steel beams

Fig. 4: Variation of the C_b coefficient with the α_{LB} parameter for castellated steel beams

Fig. 5: Variation of the C_b coefficient with the α_{LB} parameter for castellated steel beams

Fig. 6: Variation of the C_b coefficient with the α_{LB} parameter for castellated steel beams

Fig. 7: Variation of the C_b coefficient with the α_{LB} parameter for castellated steel beams

Fig. 8: Variation of the C_b coefficient with the α_{LB} parameter for castellated steel beams

Fig. 9: Variation of the C_b coefficient with the α_{LB} parameter for castellated steel beams

Table 3: Comparison between moment-gradient coefficient values recommended by international design codes

Case of loading	American code (AISC2005 [12])	European standards (EC3 [9])	Australian code (AS4100 [11])
Mid-span concentrated load	1.35	1.365	1.32

Fig.10: Variation of the C_{bFE} coefficient with the C_{Eq} factor for castellated steel beams

proposed equations and C_b values from the FE, it can be concluded that the new equations predict well the result of FE as can be seen in Fig. 10. It can be concluded that the proposed equations have good agreement compared with the FE results.

$$C_b = -0.07(\alpha_{LB})^2 + 0.11(\alpha_{LB}) + 1.3 \quad (12)$$

(for compact section)

$$C_b = -0.05(\alpha_{LB})^2 + 0.05(\alpha_{LB}) + 1.4 \quad (13)$$

(for non-compact section)

Where α_{LB} was given in Eq. 11

It can be seen that for compact and non-compact section the results of C_b in the short span have lower values than codes which defined in Table 3.

5. Conclusions

The current study investigated the elastic lateral buckling of simply supported castellated steel beams under mid-span concentrated loads. The study is conducted numerically using 3D FE model using ABAQUS software program. Variation of C_b coefficient is investigated relative to α_{LB} parameter that relates the warping rigidity to the torsional rigidity of castellated steel beams. The main conclusions that may be extracted from the study are as follows.

- From the results it can be concluded that, by increasing the values of α_{LB} the C_b values decreases. Also, the values of C_b decrease with increasing the opening size.
- Increasing of the spacing of opening to the value of ($b_4/h_w=2$) has no effect for different hole size on the reduction of C_b .
- For all hole size the value of C_b increases with increasing the value of (h_w/t_w).
- For the compact sections it was found that the values of C_b were different from the predicted code. The same observation was obtained for the non-compact sections. Hence, in this section two simplified new equations were suggested for the compact sections and non-compact sections with respect α_{LB} parameter as a variable function. By comparing the results of C_b from the proposed equations and the corresponding values from the FE, it can conclude that the new equations predict well the result of FE. Finally, it can be seen that the predicted equations have good agreement compared with the FE results.

Notations:

- H Overall height of castellated beam
 b_f Overall flange width of castellated beam
 b_1 The horizontal projection dimension of the inclined part of opening
 b_2 The horizontal dimension part of castellated opening
 b_3 The clear distance between castellated opening
 b_4 The distance between C.L of C.L of castellated opening
E Young's modulus of steel
 F_y Yield stress of steel
 F_u Ultimate stress of steel
 h_1 Half clear depth of castellated opening length of castellated beam
 t_w Web thickness of specimen
 t_f Flange thickness of specimen
 d_f The clear distance between upper flange and castellated opening
 h_o depth of opening

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