The Information System by Uncertainty Theory

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Abstract- The theories of uncertainty are very useful to treat with mathematics that needs to address and in providing a flexible model to elaborate uncertainty and vagueness involved in decision making. In this paper, we used the information system by converting the information system to the triangular fuzzy number matrix; into its membership function and by computing the relation matrices to take the suitable decision with degree. In this paper, we also introduced an application in real life.

Keywords- Fuzzy matrices, Information system, triangular fuzzy number, Horizontal Curve Design, membership function.

I. INTRODUCTION

By developing intelligent systems, we can operate with real life and engineering problems. A large part of mathematics is based on the notions of a set. Set theory, however, was founded by a single paper in 1874 by George Cantor on a property of the collection of all real numbers [1]. In classical set theory, an element either belongs to a set or does not belong to a set. Fuzziness [2] plays an essential role in human life because most of the classes encountered in the real physical world are fuzzy. In 1965, Zabeh [3] introduced the idea of a fuzzy set as an extension of the classical set theory. The idea of intuitionistic fuzzy set was first published by Krassimir T. Atanassov [4] and many works by the same author and his colleagues appeared in the literature [5, 6, 7]. Established by Florentin Smarandache in 1980, neurosophy was presented as the study of the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. The main idea was to consider an entity, "A" in relation to its opposite "Non-A". A fuzzy logic [15], [16] has extended Sanchez's approach for any application using the representation of an interval valued fuzzy matrix. They have also introduced the arithmetic mean matrix of an interval valued fuzzy matrix and directly applied Sanchez's method of the application on it. Fuzzy set theory also plays an important role in the Decision Making. Decision Making is a most important scientific, social and economic endeavor. In classical crisp decision making theories, decisions are made under conditions of certainty but in real life situations this is not possible which gives rise to fuzzy decision making theories. For decision making in fuzzy environment one may refer Bellman and Zadeh [19], [20].

In this paper, we would like to discuss how fuzzy set theory [21], [22] and fuzzy logic [23] can be used for developing knowledge-based systems using triangular fuzzy number matrices by converting the information system to fuzzy matrices to take the suitable decision making and take the decision with degree.

Definition 1.1 [24] (Triangular fuzzy number matrix)
Triangular fuzzy number matrix of order $m \times n$ is defined as

$A = (a_{ij})_{m \times n}$ where $a_{ij} = (a_{ijL}, a_{ijM}, a_{ijU})$ is the $ij$-th element of $A$. $a_{ijL}, a_{ijM}, a_{ijU}$ are the left and right spreads of $a_{ij}$ respectively and $a_{ijM}$ is the mean value.

Definition 1.2 (Maximum operation on triangular fuzzy number)
Let $A = (a_{ij})_{m \times n}$ where $a_{ij} = (a_{ijL}, a_{ijM}, a_{ijU})$ and $B = (b_{ij})_{m \times n}$ whereby

$b_{ij} = (b_{ijL}, b_{ijM}, b_{ijU})$ be two triangular fuzzy number matrices of same order. Then the maximum operation on it is given by: $I_{max} = max(A, B) = (sup(a_{ij}, b_{ij}))$, where:

$sup(a_{ij}, b_{ij}) = (sup(a_{ijL}, b_{ijL}), sup(a_{ijM}, b_{ijM}), sup(a_{ijU}, b_{ijU}))$ is the $ij$-th element of $max(A, B)$.

Definition 1.3 (Arithmetic mean (AM) for triangular fuzzy number)
Let $A = (a_1, a_2, a_3)$ be a triangular fuzzy number then, $AM = (a_1 + a_2 + a_3)/3$. The same condition holds for triangular fuzzy membership number.

II. METHODOLOGY OF SUITABLE DECISION

To find suitable decision for set of objects corresponding set of fuzzy parameters the values of membership function $\mu_{a_i}(x)$ must satisfy $0 \leq \mu_{a_i}(x) \leq 1$ and computable with formulated measure on the parameters. So, we can put the
procedure of obtained suitable decision as follows:

**Step 1:** Construct the triangle fuzzy matrices.

**Step 2:** Convert the elements of triangle fuzzy number matrix into its membership function

\[
\mu_{ij} = \frac{a_{ij} - a_{ijL}}{100} \leq \mu_{ij} \leq \frac{a_{ijU} - a_{ijL}}{100}.
\]

**Step 3:** Compute the relation matrices [18] which help the decision maker to strongly confirm the correct decision.

**Step 4:** Now, we can take the suitable decision with degree after step 3.

### III. APPLICATION OF FM IN SELECTING SPECIALIZATION

In Egypt, Faculty of Engineering, Tanta University, we suffer from lack of specialization and how and why choosing the department to each student where the famous in Egypt each student chooses the department according to that its famous and ignoring that if this student is eligible to this department or not [25]. So, we need to choose the suitable department to each student by its degree of each object where each department needs to be excellent in specific objects.

**Example 1.1**

- In electrical department need the student to be excellent in Mathematical, Physics and Computer (Logic).
- In Architecture department need the student to be excellent in drawing and Computer (Logic).
- In Mechanics department need the student to be excellent in drawing engineering, mechanics and Computer (Logic).
- In civil department need the student to be excellent in drawing engineering and Computer (Logic).

Let \( S = \{S_1, S_2, S_3, S_4\} \) be the set of students, \( D = \) (architecture, electrical, civil, mechanics) be the set of department and \( Su = \) (logic, mathematics, drawing engineering, physics, mechanics) be the set of subjects related to the departments. We assume the above students sit for examinations (i.e. over 10 marks total) on the above mentioned subjects to determine their department placements and choices. After the various examinations, the students obtained the following marks (see Table I). The departments and related subjects' requirements (see Table II).

#### TABLE I

**STUDENTS VS SUBJECTS**

<table>
<thead>
<tr>
<th></th>
<th>Logic</th>
<th>Mathematics</th>
<th>Drawing engineer</th>
<th>Physics</th>
<th>mechanics</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(8, 9, 10)</td>
<td>(8, 9, 10)</td>
<td>(5, 6, 7)</td>
<td>(8, 9, 10)</td>
<td>(4, 5, 6)</td>
</tr>
<tr>
<td>2</td>
<td>(4, 5, 6)</td>
<td>(5, 6, 7)</td>
<td>(4, 5, 6)</td>
<td>(3, 4, 5)</td>
<td>(6, 7, 8)</td>
</tr>
<tr>
<td>3</td>
<td>(6, 7, 8)</td>
<td>(5, 6, 7)</td>
<td>(6, 7, 8)</td>
<td>(4, 5, 6)</td>
<td>(3, 4, 5)</td>
</tr>
<tr>
<td>4</td>
<td>(5, 6, 7)</td>
<td>(7, 8, 9)</td>
<td>(5, 6, 7)</td>
<td>(5, 6, 7)</td>
<td>(4, 5, 6)</td>
</tr>
</tbody>
</table>

#### TABLE II

**DEPARTMENTS VS SUBJECTS**

<table>
<thead>
<tr>
<th></th>
<th>Architecture</th>
<th>Electrical</th>
<th>Civil</th>
<th>Mechanics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Logic</td>
<td>(7, 8, 9)</td>
<td>(7, 8, 9)</td>
<td>(7, 8, 9)</td>
<td>(5, 6, 7)</td>
</tr>
<tr>
<td>Math.</td>
<td>(4, 5, 6)</td>
<td>(8, 9, 10)</td>
<td>(6, 7, 8)</td>
<td>(7, 8, 9)</td>
</tr>
<tr>
<td>Draw.</td>
<td>(8, 9, 10)</td>
<td>(4, 5, 6)</td>
<td>(8, 9, 10)</td>
<td>(7, 8, 9)</td>
</tr>
<tr>
<td>Phys.</td>
<td>(4, 5, 6)</td>
<td>(7, 8, 9)</td>
<td>(4, 5, 6)</td>
<td>(4, 5, 6)</td>
</tr>
<tr>
<td>Mech.</td>
<td>(4, 5, 6)</td>
<td>(4, 5, 6)</td>
<td>(5, 6, 7)</td>
<td>(8, 9, 10)</td>
</tr>
</tbody>
</table>

### Step 1:

By converting the information system to the triangular fuzzy number matrix \((F, S)\) is another parameterized family of triangular fuzzy number matrix and gives a collection of approximate description of the subject-student in the university. Thus the triangular fuzzy number matrix \((F, S)\) represents a relation matrix \(M_{SSU}\) called student-subject matrix given by:

\[
F_1(SU_1) = \begin{bmatrix}
S_1 & (8, 9, 10) >: S_2, (4, 5, 6) >
S_2 & (6, 7, 8) >: S_1, (5, 6, 7) >
S_3 & (8, 9, 10) >: S_2, (5, 6, 7) >
S_4 & (5, 6, 7) >: S_1, (7, 8, 9) >
\end{bmatrix}
\]

\[
M_{SSU} = \begin{bmatrix}
S_1 & S_2 & S_3 & S_4 & S_5 \\
S_1 & (8, 9, 10) & (8, 9, 10) & (5, 6, 7) & (8, 9, 10) & (4, 5, 6) \\
S_2 & (4, 5, 6) & (5, 6, 7) & (4, 5, 6) & (3, 4, 5) & (6, 7, 8) \\
S_3 & (6, 7, 8) & (5, 6, 7) & (6, 7, 8) & (4, 5, 6) & (3, 4, 5) \\
S_4 & (5, 6, 7) & (7, 8, 9) & (5, 6, 7) & (5, 6, 7) & (4, 5, 6) \\
S_5 & & & & & 
\end{bmatrix}
\]

By converting the information system to the triangular fuzzy number matrix \((F, D)\) is another parameterized family of triangular fuzzy number matrix and gives a collection of approximate description of the department-subject in the university. Thus the triangular fuzzy number matrix \((F, D)\) represents a relation matrix \(M_{DSU}\) called department-subject matrix given by:

\[
F(D_1) = \begin{bmatrix}
S_1 & (7, 8, 9) >: S_2, (4, 5, 6) >: S_3, (8, 9, 10) >: S_4, (4, 5, 6) >
S_2 & (7, 8, 9) >: S_3, (4, 5, 6) >: S_1, (8, 9, 10) >
S_3 & (7, 8, 9) >: S_2, (4, 5, 6) >: S_1, (8, 9, 10) >
S_4 & (4, 5, 6) >: S_1, (5, 6, 7) >: S_3, (6, 7, 8) >: S_2, (8, 9, 10) >
S_5 & & & & 
\end{bmatrix}
\]
\[ F(D_j) = \begin{bmatrix} <S_{U_1}, (5.6.7) >; <S_{U_2}, (7.8.9) >; <S_{U_3}, (7.8.9) >; \\ <S_{U_4}, (4.5.6) >; <S_{U_5}, (8.9.10) > \end{bmatrix} \]

(2)

\[ M_{\text{arc}} = \begin{bmatrix} D_1 & D_2 & D_3 & D_4 \\ S_{U_1} & (7.8.9) & (7.8.9) & (7.8.9) & (5.6.7) \\ S_{U_2} & (4.5.6) & (8.9.10) & (6.7.8) & (7.8.9) \\ S_{U_3} & (8.9.10) & (4.5.6) & (8.9.10) & (7.8.9) \\ S_{U_4} & (4.5.6) & (7.8.9) & (4.5.6) & (4.5.6) \\ S_{U_5} & (4.5.6) & (5.6.7) & (8.9.10) & (5.6.7) \end{bmatrix} \]

Step 2:

\[ (M_{SU})_{\text{max}} = \begin{bmatrix} SU_1 & SU_2 & SU_3 & SU_4 & SU_5 \\ S_{U_1} & (0.8.0.9.1) & (0.8.0.9.1) & (0.5.0.6.0.7) & (0.8.0.9.1) & (0.4.0.5.0.6) \\ S_{U_2} & (0.4.0.5.0.6) & (0.5.0.6.0.7) & (0.4.0.5.0.6) & (0.3.0.4.0.5) & (0.6.0.7.0.8) \\ S_{U_3} & (0.6.7.0.8) & (0.5.0.6.0.7) & (0.6.0.7.0.8) & (0.4.0.5.0.6) & (0.3.0.4.0.5) \\ S_{U_4} & (0.5.0.6.0.7) & (0.7.0.8.0.9) & (0.5.0.6.0.7) & (0.5.0.6.0.7) & (0.4.0.5.0.6) \\ S_{U_5} & (0.4.0.5.0.6) & (0.4.0.5.0.6) & (0.5.0.6.0.7) & (0.8.0.9.1) \end{bmatrix} \]

(3)

Step 3:

\[ (M_{SU})_{\text{max}} = \begin{bmatrix} D_1 & D_2 & D_3 & D_4 \\ S_{U_1} & (0.7.0.8.0.9) & (0.7.0.8.0.9) & (0.7.0.8.0.9) & (0.5.0.6.0.7) \\ S_{U_2} & (0.4.0.5.0.6) & (0.8.0.9.1) & (0.6.0.7.0.8) & (0.7.0.8.0.9) \\ S_{U_3} & (0.8.0.9.1) & (0.4.0.5.0.6) & (0.8.0.9.1) & (0.7.0.8.0.9) \\ S_{U_4} & (0.4.0.5.0.6) & (0.7.0.8.0.9) & (0.4.0.5.0.6) & (0.4.0.5.0.6) \\ S_{U_5} & (0.4.0.5.0.6) & (0.4.0.5.0.6) & (0.5.0.6.0.7) & (0.8.0.9.1) \end{bmatrix} \]

(4)

\[ M_1 = (M_{SU})_{\text{max}} . (J(\cdot)(M_{SU})_{\text{max}}) = \begin{bmatrix} D_1 & D_2 & D_3 & D_4 \\ S_{U_1} & (0.7.0.8.0.9) & (0.8.0.9.1) & (0.7.0.8.0.9) & (0.7.0.8.0.9) \\ S_{U_2} & (0.4.0.5.0.6) & (0.5.0.6.0.7) & (0.6.0.7.0.8) & (0.7.0.8.0.9) \\ S_{U_3} & (0.6.0.7.0.8) & (0.6.0.7.0.8) & (0.6.0.7.0.8) & (0.6.0.7.0.8) \\ S_{U_4} & (0.5.0.6.0.7) & (0.7.0.8.0.9) & (0.6.0.7.0.8) & (0.7.0.8.0.9) \end{bmatrix} \]

Step 4:

\[ M_2 = (M_{SU})_{\text{max}} . (J(\cdot)(M_{SU})_{\text{max}}) = \begin{bmatrix} D_1 & D_2 & D_3 & D_4 \\ S_{U_1} & (0.6.0.5.0.4) & (0.6.0.5.0.4) & (0.6.0.5.0.4) & (0.6.0.5.0.4) \\ S_{U_2} & (0.6.0.5.0.4) & (0.6.0.5.0.4) & (0.6.0.5.0.4) & (0.6.0.5.0.4) \\ S_{U_3} & (0.4.0.5.0.4) & (0.4.0.5.0.4) & (0.4.0.5.0.4) & (0.4.0.5.0.4) \\ S_{U_4} & (0.6.0.5.0.4) & (0.6.0.5.0.4) & (0.6.0.5.0.4) & (0.6.0.5.0.4) \end{bmatrix} \]

Step 5:

\[ M_3 = (J(\cdot)(M_{SU})_{\text{max}}) . (J(\cdot)(M_{SU})_{\text{max}}) = \begin{bmatrix} D_1 & D_2 & D_3 & D_4 \\ S_{U_1} & (0.5.0.5.0.4) & (0.5.0.5.0.4) & (0.5.0.5.0.4) & (0.5.0.5.0.4) \\ S_{U_2} & (0.6.0.5.0.4) & (0.6.0.5.0.4) & (0.6.0.5.0.4) & (0.6.0.5.0.4) \\ S_{U_3} & (0.4.0.5.0.4) & (0.4.0.5.0.4) & (0.5.0.6.0.5) & (0.7.0.6.0.5) \\ S_{U_4} & (0.5.0.5.0.4) & (0.5.0.5.0.4) & (0.5.0.5.0.4) & (0.6.0.5.0.4) \end{bmatrix} \]

(5)

(6)

\[ M_4 = (J(\cdot)(M_{SU})_{\text{max}}) . (J(\cdot)(M_{SU})_{\text{max}}) = \begin{bmatrix} D_1 & D_2 & D_3 & D_4 \\ S_{U_1} & (0.6.0.5.0.4) & (0.6.0.5.0.4) & (0.6.0.5.0.4) & (0.6.0.5.0.4) \\ S_{U_2} & (0.6.0.5.0.4) & (0.6.0.5.0.4) & (0.6.0.5.0.4) & (0.6.0.5.0.4) \\ S_{U_3} & (0.6.0.5.0.4) & (0.6.0.5.0.4) & (0.6.0.5.0.4) & (0.6.0.5.0.4) \end{bmatrix} \]

(7)

Step 4:

\[ D_1 = \begin{bmatrix} 0.03 \\ -0.07 \\ 0.21 \\ 0.1 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0.1 \\ 0.07 \\ 0.23 \\ 0.3 \end{bmatrix}, \quad D_3 = \begin{bmatrix} 0.03 \\ 0.17 \\ 0.33 \\ 0.3 \end{bmatrix}, \quad D_4 = \begin{bmatrix} 0.03 \\ 0.17 \\ 0.33 \\ 0.3 \end{bmatrix} \]

(8)

S\text{1} \text{ can select electrical (Electrical Engineer), mechanics \text{(Mechanical Engineer)} or architecture (architecture Engineer) and by the requesting department or the biggest degree chosen this student in the suitable department. S2 is to read mechanics (Mechanical Engineer), S3 is to read architecture (architecture Engineer) or civil (civil Engineer) and by the requesting department or the biggest degree chosen this student in the suitable department. And S4 is to read civil (civil Engineer). So, we deduce that this case study presented in this mark can be applied in many real life applications. For example: political or social case.}
Let $\mathcal{X} = \{ (x, y) : (x, y) \in \mathcal{R} \text{ such that } \mu(y, y) \geq \alpha \}$, then 

$$S^A_{\alpha} = \{ (x, y) : (x, y) \in \mathcal{R} \text{ and } S^A_{\alpha} \subseteq \mathcal{R} \} $$

Example 3.1: If $\alpha \geq 0.8$ be $V = \{ 00000000000 \}$ and $V_1 = \{ 11111111111 \}$ and when $\alpha \geq 0.9$ be $V = \{ 00000000000 \}$ and $V_1 = \{ 00000000000 \}$, as given by (see Table III).

Using the above definitions, we obtain the following approximations:

$$S^A_{\alpha} = \{ (x, y) : (x, y) \in \mathcal{R} \text{ and } S^A_{\alpha} \subseteq \mathcal{R} \} $$

$$S^A_{\alpha} = \{ (x, y) : (x, y) \in \mathcal{R} \text{ and } S^A_{\alpha} \subseteq \mathcal{R} \} $$

Definition 1.6: Let $A, B, C$ be fuzzy sets and 

$$\mu_A(x) \leq \mu_B(y), \mu_C(z)$$

be a membership of $x \in A, y \in B, z \in C$ then the $\alpha - \text{level decision classification of } A$ can be defined as:

$$D_\alpha = \{ (x, y) : (x, y, z) \in A \times B \times C \text{ and } \mu_A(x) \geq \mu_B(y) \geq \mu_C(z) \}$$

$$D^A = \{ (x, y) : (x, y) \in A \}$$

Example 1.2

Let $X = \{ x : V_1 \geq V = \text{Yes} \}$, as given by Table III. In fact, the set $X$ consists of three objects: $R_1, R_2, R_3, R_4, R_5, R_6, R_7, R_8, R_9, R_{10}, R_{11}$.

Now, we want to describe this set in terms of the set of conditional attributes $A = \{ \text{suitable or not} \}$.

V. NEW APPROXIMATION OPERATION ON HORIZONTAL CURVE DESIGN

In this paper, we also used lower and upper approximations of above set are important to reduce the number of sets to make suitable decision. If we used uncertainty concepts to determine any set, we must find two sets one of them included and other contained in this set. Under classification of set of parameter and condition, we formed four approximations operators as follows:

Definition 1.7: Let $A_1, A_2, \ldots, A_n$ be fuzzy sets of comparison values with set $A$ then comparison lower approximation of set $A_i$ can be defined as:

$$L^A_{\alpha}(X) = \{ k : k \in S^A_{\alpha} \} $$

and comparison upper approximation of set $A_i$ can be defined as:

$$U^A_{\alpha}(X) = \{ k : k \in S^A_{\alpha} \} $$

where $X \subseteq \mathcal{U}, \mathcal{U}$ is universal set.

Definition 1.8: Let $A_1, A_2, \ldots, A_n$ be fuzzy sets of comparison values with set $A$ then decision lower approximation of $X \subseteq \mathcal{U} \cup \mathcal{U}$ is universal set can be defined as:

$$L^A_{\alpha}(X) = \{ k : k \in D^A_{\alpha} \} $$

and decision upper approximation of $X \subseteq \mathcal{U} \cup \mathcal{U}$ is universal set can be defined as:

$$U^A_{\alpha}(X) = \{ k : k \in D^A_{\alpha} \} $$

Using the above definitions, we obtain the following approximations: Case 1: At $\alpha \geq 0.8$ the comparison lower approximation $L^A_{\alpha}(X) = \emptyset$, the comparison upper approximation $U^A_{\alpha}(X) = \{ R_1, R_2, R_3, R_4, R_5, R_6, R_7, R_8, R_9, R_{10}, R_{11} \}$ The decision lower approximation $L^A_{\alpha}(X) = \{ R_1, R_2 \}$ and the decision upper approximation $U^A_{\alpha}(X) = \{ R_1, R_2, R_3, R_4, R_5, R_6, R_7, R_8, R_9, R_{10}, R_{11} \}$

Case 2: At $\alpha \geq 0.9$, the comparison lower approximation $L^A_{\alpha}(X) = \{ R_1, R_2, R_3, R_4 \}$, the comparison upper approximation $U^A_{\alpha}(X) = \{ R_1, R_2, R_3, R_4, R_5, R_6, R_7, R_8, R_9, R_{10}, R_{11} \}$

The decision lower approximation $L^A_{\alpha}(X) = \{ R_1, R_2 \}$ and the decision upper approximation $U^A_{\alpha}(X) = \{ R_1, R_2, R_3, R_4, R_5, R_6, R_7, R_8, R_9, R_{10}, R_{11} \}$

One can easily the following properties of approximations. Proposition 1.1 Let $L^A_{\alpha}(X)$ and $L^A_{\alpha}(X)$ be comparison and decision lower approximation then the following are satisfied.

i. $L^A_{\alpha}(\emptyset) = \emptyset.$

Proof:

Let $L^A_{\alpha}(X) = \emptyset$. If $X = \emptyset$, then $L^A_{\alpha}(\emptyset) = \emptyset.$

By the same way, $L^A_{\alpha}(\emptyset) = \emptyset.$

ii. $L^A_{\alpha}(\mathcal{U}) = \mathcal{U}.$

Proof:

Let $L^A_{\alpha}(X) = \mathcal{U}$. If $X = \mathcal{U}$, then $L^A_{\alpha}(\mathcal{U}) = \mathcal{U}.$

By the same way, $L^A_{\alpha}(\mathcal{U}) = \mathcal{U}.$

iii. $L^A_{\alpha}(X \cap Y) \subseteq L^A_{\alpha}(X) \cap L^A_{\alpha}(Y).$

Proof:

Let $x \in L^A_{\alpha}(X) \cap L^A_{\alpha}(Y)$ implies $x \in L^A_{\alpha}(X)$ then $k \leq x \in L^A_{\alpha}(Y)$ and $k \leq x \in L^A_{\alpha}(Y)$.

By the same way, $L^A_{\alpha}(X \cap Y) \subseteq L^A_{\alpha}(X) \cap L^A_{\alpha}(Y).$

iv. $X \subseteq Y$ implies $L^A_{\alpha}(X) \subseteq L^A_{\alpha}(Y).$

Proof:

If $x \in L^A_{\alpha}(X) \subseteq Y$, then $x \in L^A_{\alpha}(X)$.

By the same way, $X \subseteq Y$ implies $L^A_{\alpha}(X) \subseteq L^A_{\alpha}(Y).$

v. $L^A_{\alpha}(X \cap Y) \subseteq L^A_{\alpha}(X) \cup L^A_{\alpha}(Y).$

Proof:

If $x \in L^A_{\alpha}(X) \cap L^A_{\alpha}(Y)$ implies $x \in L^A_{\alpha}(X) \cup L^A_{\alpha}(Y)$.

By the same way, $L^A_{\alpha}(X \cap Y) \subseteq L^A_{\alpha}(X) \cup L^A_{\alpha}(Y).$
Thus $x \in L_{V_1}^1 (X \cup Y)$. It is easy to notice that the reverse implication is not true in general, for $L_{V_1}^1 (X \cup Y) \neq L_{V_1}^1 (X) \cup L_{V_1}^1 (Y)$.

By the same way, $L_{V_1}^2 (X \cup Y) \equiv L_{V_1}^2 (X) \cup L_{V_1}^2 (Y)$.

Proposition 1.2 Let $U_{V_1}^1 (X)$ and $U_{V_1}^2 (X)$ be comparison and decision upper approximation then the following are satisfied.

i. $U_{V_1}^1 (\phi) = \phi$.

ii. $U_{V_1}^1 (U) = U$.

iii. $U_{V_1}^1 (X \cup Y) = U_{V_1}^1 (X) \cup U_{V_1}^1 (Y)$.

iv. $X \subseteq Y$ implies $U_{V_1}^1 (X) \subseteq U_{V_1}^1 (Y)$.

v. $U_{V_1}^1 (X \cap Y) \subseteq U_{V_1}^1 (X) \cap U_{V_1}^1 (Y)$.

Proposition 1.3 Let $L_{V_1}^1 (X)$ and $L_{V_1}^2 (X)$ be comparison and decision lower approximation, $L_{V_1}^1 (X) \hspace{1mm} \text{and} \hspace{1mm} U_{V_1}^2 (X)$ be comparison and decision upper approximation then the following are satisfied.

i. $L_{V_1}^1 (X) \subseteq X \subseteq U_{V_1}^1 (X)$.

Proof:

If $x \in X \Rightarrow x \in L_{V_1}^1 (X)$ and $x \in U_{V_1}^1 (X)$.

But $L_{V_1}^1 (X) \subseteq X$ and $X \subseteq U_{V_1}^1 (X)$.

Then $L_{V_1}^1 (X) \subseteq X \subseteq U_{V_1}^1 (X)$.

By the same way, $L_{V_1}^2 (X) \subseteq X \subseteq U_{V_1}^2 (X)$.

ii. $L_{V_1}^1 (U - X) = U - U_{V_1}^1 (X)$.

Proof:

If $x \in (U - X) \Leftrightarrow x \in L_{V_1}^1 (U - X)$

So $x \in (U - X) \Leftrightarrow x \in U - U_{V_1}^1 (X) \Leftrightarrow x \notin U_{V_1}^1 (X) \Leftrightarrow \text{non} \hspace{1mm} x \in U_{V_1}^1 (X)$

Then $L_{V_1}^1 (U - X) = U - U_{V_1}^1 (X)$.

By the same way, $L_{V_1}^2 (U - X) = U - U_{V_1}^2 (X)$.

iii. $U_{V_1}^1 (U - X) = U - U_{V_1}^1 (X)$.

Proof:

If $x \in (U - X) \Leftrightarrow x \in U_1 (U - X)$

So $x \in (U - X) \Leftrightarrow x \in U - L_{V_1}^1 (X) \Leftrightarrow x \notin L_{V_1}^1 (X) \Leftrightarrow \text{non} \hspace{1mm} x \in L_{V_1}^1 (X)$

Then $U_{V_1}^1 (U - X) = U - L_{V_1}^1 (X)$.

By the same way, $U_{V_1}^2 (U - X) = U - L_{V_1}^2 (X)$.

iv. $L_{V_1}^1 (L_{V_1}^1 (X)) = U_{V_1}^1 (L_{V_1}^1 (X)) = L_{V_1}^1 (X)$.

Proof:

Let $L_{V_1}^1 (X) = X$ and $U_{V_1}^1 (X) = X$. If $X = L_{V_1}^1 (X)$

Then $L_{V_1}^1 (L_{V_1}^1 (X)) = U_{V_1}^1 (L_{V_1}^1 (X)) = L_{V_1}^1 (X)$.

By the same way, $L_{V_1}^2 (L_{V_1}^2 (X)) = U_{V_1}^2 (L_{V_1}^2 (X)) = L_{V_1}^2 (X)$.

v. $U_{V_1}^1 (U_{V_1}^1 (X)) = L_{V_1}^1 (U_{V_1}^1 (X)) = U_{V_1}^1 (X)$.

Proof:

Let $L_{V_1}^1 (X) = X$ and $U_{V_1}^1 (X) = X$. If $X = L_{V_1}^1 (X)$

Then $L_{V_1}^1 (U_{V_1}^1 (X)) = U_{V_1}^1 (L_{V_1}^1 (X)) = U_{V_1}^1 (X)$.

By the same way, $L_{V_1}^2 (U_{V_1}^2 (X)) = U_{V_1}^2 (L_{V_1}^2 (X)) = U_{V_1}^2 (X)$.

Where $-X$ denotes $U - X$.

It is easily seen that the lower and the upper approximations of a set, are respectively, the interior and the closure of this set in the topology generated by the indiscernibility relation.

Since any set can be measure by upper and lower approximations, which depends on the method of classifications by parameters, So, it is important to introduce the various kinds of Rough notations on sets.

Definition 1.8 Let $U = \{R_1, R_2, \ldots, R_m\}$ be set of objects and $V_i = \{V_1^i, V_2^i, \ldots, V_m^i\}$ be set of comparison fuzzy parameters, then the following four categories of vagueness defined as:

1. A set $X$ is roughly $V_1$-definable, iff $L_{V_1}^1 (X) \neq \phi$ and $U_{V_1}^1 (X) \neq U$.

2. A set $X$ is internally $V_1$-undefinable, iff $L_{V_1}^1 (X) = \phi$ and $U_{V_1}^1 (X) \neq U$.

3. A set $X$ is externally $V_1$-undefinable, iff $L_{V_1}^1 (X) \neq \phi$ and $U_{V_1}^1 (X) = U$.

4. A set $X$ is totally $V_1$-undefinable, iff $L_{V_1}^1 (X) = \phi$ and $U_{V_1}^1 (X) = U$.

Definition 1.9 Let $U = \{R_1, R_2, \ldots, R_m\}$ be set of objects and $V = \{V_1, V_2, \ldots, V_m\}$ be a set of standard fuzzy parameters defined on $U$ the following four categories of vagueness defined as:

1. A set $X$ is roughly $V$-definable, iff $L_{V}^1 (X) \neq \phi$ and $U_{V}^1 (X) \neq U$.

2. A set $X$ is internally $V$-undefinable, iff $L_{V}^1 (X) = \phi$ and $U_{V}^1 (X) \neq U$.

3. A set $X$ is externally $V$-undefinable, iff $L_{V}^1 (X) \neq \phi$ and $U_{V}^1 (X) = U$.

4. A set $X$ is totally $V$-undefinable, iff $L_{V}^1 (X) = \phi$ and $U_{V}^1 (X) = U$.

The intuitive meaning of this classification is the following:

- A set $X$ is roughly $V$-definable means that with the help of $A$ we are able to decide for some elements of $U$ that they belong to $X$ and for some elements of $U$ that they belong to $-X$.

- A set $X$ is internally $V$-undefinable means that using $A$ we are able to decide for some elements of $U$ that they belong to $-X$, but we are unable to decide for any element of $U$ whether it belong to $X$. 


A set $X$ is externally $\mathcal{V}$-undefined means that using $A$ we are able to decide for some elements of $\mathcal{U}$ that they belong to $X$, but we are unable to decide for any element of $\mathcal{U}$ whether it belongs to $-X$.

A set $X$ is totally $\mathcal{V}$-undefined means that using $A$ we are unable to decide for any element of $\mathcal{U}$ whether it belongs to $X$ or $-X$.

A rough set can be also characterized numerically by the following coefficient:

$$\alpha_A^1(X) = \left[ \frac{L_A^1(X)}{U^1_\mathcal{V}(X)} \right], \quad \alpha_A^{12}(X) = \left[ \frac{L_A^{12}(X)}{U^{12}_\mathcal{V}(X)} \right],$$

And by the same way, $\alpha_A^{22}(X) = \left[ \frac{L_A^{22}(X)}{U^{22}_\mathcal{V}(X)} \right]$ called the accuracy of approximation, where $X$ denotes the cardinality of $X \neq \emptyset$. Obviously $0 \leq \alpha_A(X) \leq 1$. If $\alpha_A(X) = 1$, $X$ is crisp with respect to $A$ ($X$ is precise with respect to $A$), and otherwise, if $\alpha_A(X) < 1$, $X$ is rough with respect to $A$ ($X$ is vague with respect to $A$).

VI. SUMMARY

We review several concepts which will be used in this paper; the development from fuzzy set to its extensions for instance, the neutrosophic set, the intuitionistic set, the intuitionistic fuzzy set and display in this paper the fuzzy matrix that is quite interesting and useful in many application areas. We also display an application associate with it.

VII. CONCLUSION

The case study presented in this work can be applied in many real life applications. For example: medicine, political or social cases.

REFERENCES