Topological approximations based on relations induced from differences

Rukaia M. Rashed

*Department of Mathematics, Faculty of Education, Al-Zawia University,
Libya*

A. A. azzam

*Department of Mathematics, Faculty of Science, Assuit University, New
Valley, Egypt*

Abstract

The starting point of rough set theory is the indiscernibility relation, which is generated by information about objects of interest. The indiscernibility relation is intended to express the fact that due to the lack of knowledge we are unable to discern some objects employing the available information. Therefore, generally, we are unable to deal with single object. Nevertheless, we have to consider clusters of indiscernible objects.

In this paper, a relationship was created through a table of given data. This relationship is based on the expert's opinion in the field of data. Based on this relationship, we constructed a topological under the lowest possible conditions. This topology describes the roughness of the set and shows that the intermittent topology obtained is stronger and more precise than others. Other studies, also in this paper, compared the approximations obtained using the precision scale to determine the best.

Introduction

Rough set theory, proposed by Pawlak in 1980's as a result of a long term program on mathematical fundamental research can be seen as a new mathematical approach to vagueness (set) and uncertainty (element) from databases (information systems) [3]. One of the important issues in rough set-based database analysis is discovering dependencies between attributes. An attribute dependency states that the value of an attribute is uniquely determined by the values of some other attributes. The objective of discovering attribute dependency is to find the relationship among attributes in information systems. A rough set model in databases may therefore be considered as a method for constructing a topological space using indiscernibility relation on the set of objects [2]. In a reverse process, we can generalize the notions of rough sets based on the topological space.

Definable sets, *i.e.*, a family of union of one or more equivalence classes are substituted by open sets in defining the lower approximations, and by closed sets in defining upper approximations, as the basic concepts of rough set theory.

In this paper, we focus on the dependency of attributes in information systems. We show that a dependency of attributes in information systems determines a finer quasi-discrete topology which can be considered as a quasi-discrete topological subspace. Then, we use the notion of the topology to characterize the roughness of a set. The rest of this paper is organized as follows. Section 3 describes the rough set theory in information systems. Section 4 describes the notion of topological space and several topological aspects of sets.

Note

In each part of this paper we used the same example data.

1- Preliminaries

The concept of topological space structures is one the most powerful notions in system analysis. The notions of topological space, open set and closed set, closure of a set, interior of a set, exterior of a set, boundary of a set and base of a topological space has been handled in [1, 7].

Example 1.1

Suppose that data about 6 students is given, as shown in table (1)

Student	Analysis	Algebra	Statistics	Decision
1	bad	good	medium	accept
2	good	bad	medium	accept
3	good	good	good	accept
4	bad	good	bad	reject
5	good	bad	medium	reject
6	bad	good	good	accept

Table (1)

From table (1) we have

$U = \{1,2,3,4,5,6\}$, $A = \{\text{Analysis, Algebra, Statistics, Decision}\}$, where

$C = \{\text{Analysis, Algebra, Statistics}\}$, $D = \{\text{Decision}\}$ &

$V_{\text{Analysis}} = \{\text{bad, good}\}$, $V_{\text{Algebra}} = \{\text{bad, good}\}$,

$V_{\text{Statistics}} = \{\text{bad, medium, good}\}$ & $V_{\text{Decision}} = \{\text{accept, reject}\}$, in the information system $S = (U, A, V, f)$ where U is the universe, A is the set of attributes, $V = \bigcup_{a \in I} V_a$, V_a is the domain (value set) of attribute a , $f: U \times A \rightarrow V$ is a total function such that $f(u, a) \in V$, for every $(u, a) \in U \times A$, called information (knowledge) function.

2- Indiscernibility relation

From table (1), it is noted that students 2, 3 & 5 are indiscernible with respect to the attribute Analysis, meanwhile, students 3& 6 are indiscernible with respect to the attribute Algebra and Decision, and students 2 & 5 are indiscernible with respect to the attribute Analysis, Algebra and Statistics. Now I will make the schedule of the relation as the following, from the previous table we determine the number of times between element until the group ends then we get a symmetrical table the elements of its diameter are zeros.

	1	2	3	4	5	6
1	0	2	2	2	3	1
2	2	0	2	4	1	3
3	2	2	0	3	3	1
4	2	4	3	0	3	2
5	3	1	3	3	0	4
6	1	3	1	2	4	0

Table (2)

Definition 2.1[8, 9]

Let $S = (U, A, f)$ be an information system and let B be any subset of A . Two elements $x, y \in U$ are be B –indiscernible (indiscernible by the set of attribute $B \subseteq A$ in S) if and only if $f(x, a) = f(y, a)$, for every $a \in B$.

Definition 2.1

Let R be a general relation over U , and $S = (U, A, V, f)$, be an information system, B any subset of A then the minimal a construction set is the family of all after sets of R containing an element $x \in U$, and denoted by $(m. s)$ where $(m. s) = \{xR: x \in U\}$ & $xR = \{y: xRy, y \in U\}$.

Now we conclusion the a after sets or before sets with the all numbers of the table (2) as follows

With respect to a relationship that yields results less than 2

$$1R0 = \{1,6\}, 2R0 = \{2,5\}, 3R0 = \{3,6\}, 4R0 = \{4\}, 5R0 = \{2,5\},$$

$$6R0 = \{1,3,6\} \Rightarrow R_1 = \{(1,1), (1,6), (2,2), (2,5), (3,3), (3,6), (4,4), (5,2), (5,5), (6,1), (6,3), (6,6)\}$$

The first after set is

$$S_1 = \{\{4\}, \{1,6\}, \{3,6\}, \{2,5\}, \{1,3,6\}\}$$

With respect to a relationship that yields results less than 3

$$1R1 = \{1,2,3,4,6\}, 2R1 = \{1,2,3,5\}, 3R1 = \{1,2,3,6\}, 4R1 = \{1,4,6\},$$

$$5R1 = \{2,5\}, 6R1 = \{1,3,4,6\}$$

$$\Rightarrow R_2 = \{(1,1), (1,2), (1,3), (1,4), (1,6), (2,1), (2,2), (2,3), (2,5), (3,1), (3,2), (3,3), (3,6), (4,1), (4,4), (4,6), (5,2), (5,5), (6,1), (6,3), (6,4), (6,6)\}$$

The second after set is $S_2 =$

$$\{\{1,2,3,4,6\}, \{1,2,3,5\}, \{1,2,3,6\}, \{1,4,6\}, \{2,5\}, \{1,3,4,6\}\}.$$

With respect to a relationship that yields results less than 4

$$1R3 = U, 2R3 = \{1,2,3,5,6\}, 3R3 = U, 4R3 = \{2,3,4,5,6\}, 5R3 = \{1,2,3,4,5\},$$

$$6R3 = \{1,2,3,4,6\}$$

$$\Rightarrow R_3 = \left\{ \begin{array}{l} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,5), (2,6), \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,2), (4,3), (4,4), (4,5), (4,6), \\ (5,1), (5,2), (5,3), (5,4), (5,5), (6,1), (6,2), (6,3), (6,4), (6,6) \end{array} \right\}$$

The third after set is $S_3 = \{U, \{1,2,3,5,6\}, \{1,2,3,4,5\}, \{1,2,3,4,6\}, \{2,3,4,5,6\}\}.$

Now we define minimal a construction $(m.s)$ as

$$(m.s)_1 = \{\{4\}, \{1,6\}, \{3,6\}, \{2,5\}, \{1,3,6\}\},$$

$$(m.s)_2 = \{\{1,2,3,4,6\}, \{1,2,3,5\}, \{1,2,3,6\}, \{1,4,6\}, \{2,5\}, \{1,3,4,6\}\},$$

$$(m.s)_3 = \{U, \{1,2,3,5,6\}, \{2,3,4,5,6\}, \{1,2,3,4,5\}, \{1,2,3,4,6\}\},$$

$$\text{Also, } (m.s)_1^c = \{\{2,3,4,5\}, \{1,3,4,6\}, \{1,2,4,5\}, \{1,2,3,5,6\}, \{2,4,5\}\},$$

$$(m.s)_2^c = \{\{5\}, \{4,6\}, \{4,5\}, \{2,3,5\}, \{1,3,4,6\}, \{2,5\}\},$$

$$(m.s)_3^c = \{\emptyset, \{4\}, \{1\}, \{6\}, \{5\}\}$$

3- Rough set approximations

Rough set theory [4] proposes a new mathematical approach to imperfect knowledge, i.e. to vagueness (or imprecision). In this approach vagueness is expressed by region of a set.

Rough set concept can be defined by means of topological operations, interior and closure, called approximations.

Definition 3.1[4, 5]

Let $X \subseteq U$ and R be an equivalence relation, then we denote the boundary of X in U as $BND_R(X) = \overline{R}(X) - \underline{R}(X)$ and,

- 1- X is an exact if $\underline{R}(X) = \overline{R}(X) \Rightarrow BND_R(X) = \Phi$.
- 2- If $BND_R(X) \neq \Phi$ then $\underline{R}(X) \subset X \subset \overline{R}(X)$, the set X is called a rough set.

Definition 3.1

Let R be a general relation over U , and $S = (U, A, V, f)$, be an information system, B any sub set of A & X is any subset of U , then the B – lower approximation of X denoted by $\underline{B}(X)$ and B – upper approximation of X denoted by $\overline{B}(X)$, respectively are defined by

$$\underline{B}(X) = \cup\{x \in U: (m.s) \subseteq X\}, \quad \overline{B}(X) = \cap\{x \in U: (m.s) \cap X \neq \emptyset\}.$$

From above definition we obtained the results

- 1- The boundary region $BND_B(X) = \overline{B}(X) - \underline{B}(X)$.
- 2- The positive region $POS_B(X) = \underline{B}(X)$.
- 3- The negative region $NEG_B(X) = U - \overline{B}(X)$.

X is said to be R – exact if $\underline{R}(X) = \overline{R}(X)$ otherwise X is said to be R – rough.

Now, in this part we will generate a topological space and we will find the minimal construction for all the relationships as in the following example where we can find a topology and its base as follows:

Example 3.1

From example 1.1, we have:

$$R_1 = \{(1,1), (1,6), (2,2), (2,5), (3,3), (3,6), (4,4), (5,2), (5,5), (6,1), (6,3), (6,6)\}, \quad \frac{U}{R_1} = \{\{4\}, \{1,6\}, \{3,6\}, \{2,5\}, \{1,3,6\}\},$$

the minimal base for T_1 is $\beta_1 = \{U, \emptyset, \{4\}, \{6\}, \{1,6\}, \{3,6\}, \{2,5\}, \{1,3,6\}\}.$

$$T_1 = \left\{ \begin{array}{l} U, \emptyset, \{4\}, \{1,6\}, \{3,6\}, \{2,5\}, \{4,6\}, \{1,3,6\}, \{1,4,6\}, \\ \{6\}, \{2,4,5\}, \{2,5,6\}, \{1,3,4,6\}, \{1,2,5,6\}, \{2,3,5,6\}, \\ \{3,4,6\}, \{2,4,5,6\}, \{1,2,3,5,6\}, \{1,2,4,5,6\}, \{2,3,4,5,6\} \end{array} \right\},$$

$$T_1^c = \left\{ \begin{array}{l} U, \emptyset, \{1,2,3,5,6\}, \{2,3,4,5\}, \{1,2,4,5\}, \{1,3,4,6\}, \\ \{1\}, \{3\}, \{4\}, \{1,2,3,5\}, \{2,4,5\}, \{2,3,5\}, \{1,2,3,4,5\}, \\ \{1,3,4\}, \{1,3,6\}, \{2,5\}, \{3,4\}, \{1,4\}, \{1,2,5\}, \{1,3\} \end{array} \right\}$$

It is obvious that $\emptyset$ & U are closed and open (clopen) sets, Also the sets $\{1,3,6\}$ & $\{2,5\}$ are clopen sets, the set $\{1,4,6\}$ is open, the set $\{1\}$ is closed and the set $\{2,5\}$ is neither open nor closed.

Table (3) summarizing interior, exterior, closure and boundary of sets with respect to T_1

Set	Interior	Exterior	Closure	Boundary
$\{5\}$	$\emptyset$	$\{1,3,4,6\}$	$\{2,5\}$	$\{2,5\}$
$\{1,3,4\}$	$\{4\}$	$\{2,5,6\}$	$\{1,3,4\}$	$\{1,3\}$
$\{2,3,4\}$	$\{4\}$	$\{1,6\}$	$\{2,3,4,5\}$	$\{2,3,5\}$
$\{1,2,5,6\}$	$\{1,2,5,6\}$	$\{4\}$	$\{1,2,3,5,6\}$	$\{3\}$

Table (3)

With respect to T_2

$$R_2 = \{(1,1), (1,2), (1,3), (1,4), (1,6), (2,1), (2,2), (2,3), (2,5), (3,1), (3,2), \} \frac{U}{R_2} =$$

$\{\{1,2,3,4,6\}, \{1,2,3,5\}, \{1,2,3,6\}, \{1,4,6\}, \{2,5\}, \{1,3,4,6\}\}$, the minimal base for T_2 is

$$\beta_2 = \{U, \emptyset, \{1\}, \{2\}, \{1,3\}, \{1,6\}, \{2,5\}, \{1,2,3\}, \{1,3,6\}, \{1,2,3,5\}, \{1,2,3,6\}, \{1,4,6\}, \{1,3,4,6\}, \{1,2,3,4,6\}\}$$

$$T_2 = \{U, \emptyset, \{1\}, \{2\}, \{1,2\}, \{1,3\}, \{1,6\}, \{2,5\}, \{1,2,3\}, \{1,2,5\}, \{1,2,3,5\}, \{1,2,3,4,6\}, \{1,2,6\}, \{1,3,6\}, \{1,4,6\}, \{1,2,3,6\}, \{1,2,4,6\}, \{1,2,5,6\}, \{1,3,4,6\}, \{1,2,3,5,6\}\}$$

$$T_2^c = \{U, \emptyset, \{4\}, \{5\}, \{2,5\}, \{3,4\}, \{3,5\}, \{4,5\}, \{4,6\}, \{2,4,5\}, \{2,3,5\}, \{3,4,5\}, \{3,4,6\}, \{4,5,6\}, \{3,4,5,6\}, \{2,4,5,6\}, \{2,3,4,5\}, \{1,3,4,6\}, \{1,3,4,5,6\}, \{2,3,4,5,6\}\}$$

It is obvious that $\emptyset$ & U are closed and open (clopen) sets, Also the sets $\{2,5\}$ & $\{1,3,4,6\}$ are clopen sets, the set $\{1,4,6\}$ is open, the set $\{5\}$ is closed and the set $\{1,3,4\}$ is neither open nor closed.

Table (4) summarizing interior, exterior, closure and boundary of sets with respect to T_2

Set	Interior	Exterior	Closure	Boundary
$\{5\}$	$\emptyset$	$\{1,2,3,4,6\}$	$\{5\}$	$\{5\}$
$\{1,3,4\}$	$\emptyset$	$\{2,5\}$	$\{1,3,4,6\}$	$\{1,3,4,6\}$
$\{2,3,4\}$	$\emptyset$	$\emptyset$	U	U
$\{1,2,5,6\}$	$\{2,5\}$	$\emptyset$	U	$\{1,3,4,6\}$

Table (4)

With respect to T_3

$$R_3 = \left\{ \begin{array}{l} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,5), (2,6), \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,2), (4,3), (4,4), (4,5), (4,6), \\ (5,1), (5,2), (5,3), (5,4), (5,5), (6,1), (6,2), (6,3), (6,4), (6,6) \end{array} \right\},$$

$$\frac{U}{R_3} = \{U, \{1,2,3,5,6\}, \{1,2,3,4,5\}, \{1,2,3,4,6\}, \{2,3,4,5,6\}\}, \text{ the minimal base for } T_2 \text{ is}$$

$$\beta_3 = \{U, \emptyset, \{1\}, \{2\}, \{3\}, \{4\}, \{1,2\}, \{1,3\}, \{1,6\}, \{2,3,4\}\}.$$

$$T_3 = \left\{ \begin{array}{l} U, \emptyset, \{1\}, \{2\}, \{4\}, \{1,2\}, \{1,3\}, \{1,4\}, \{1,6\}, \{2,3\}, \{2,4\}, \\ \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,2,6\}, \{1,3,4\}, \{1,3,6\}, \{1,4,6\}, \\ \{2,3,4\}, \{1,2,3,4\}, \{1,3,4,6\}, \{1,2,4,6\}, \{3,4,5\} \end{array} \right\},$$

$$T_3^c = \left\{ \begin{array}{l} U, \emptyset, \{2,5\}, \{3,5\}, \{5,6\}, \{1,2,6\}, \{1,5,6\}, \{2,3,5\}, \{2,4,5\}, \{2,5,6\}, \\ \{3,4,5\}, \{3,5,6\}, \{4,5,6\}, \{1,2,5,6\}, \{1,3,5,6\}, \{1,4,5,6\}, \{2,3,4,5\}, \\ \{2,3,5,6\}, \{2,4,5,6\}, \{3,4,5,6\}, \{1,2,3,5,6\}, \{1,3,4,5,6\}, \{2,3,4,5,6\} \end{array} \right\},$$

It is obvious that $\emptyset$ & U are closed and open (clopen) sets, Also the sets $\{1,2,6\}$ & $\{3,4,5\}$ are clopen sets, the set $\{1,3,6\}$ is open, the set $\{5,6\}$ is closed and the set $\{5\}$ is neither open nor closed.

Table (5) summarizing interior, exterior, closure and boundary of sets with respect to T_3

Set	Interior	Exterior	Closure	Boundary
$\{5\}$	$\emptyset$	$\{1,2,3,4,6\}$	$\{2,5\}$	$\{2,5\}$
$\{1,3,4\}$	$\{1,3,4\}$	$\{2\}$	$\{1,3,4,5,6\}$	$\{5,6\}$
$\{2,3,4\}$	$\{2,3,4\}$	$\{1,6\}$	$\{2,3,4,5\}$	$\{5\}$
$\{1,2,5,6\}$	$\{1,2,6\}$	$\{3,4\}$	$\{1,2,5,6\}$	$\{5\}$

Table (5)

Also we have $\mathcal{R} = R_1 \cap R_2 \cap R_3 = R_1$ & $T = T_1$

Definition 3.2

The accuracy of approximation (accuracy of roughness) of any subset $X \subseteq U$, with respect to $B \subseteq A$ denoted by $\alpha_B(X)$ is measured by $\alpha_B(X) = \left| \frac{\underline{R}_B(X)}{\overline{R}_B(X)} \right|$.

Example 3.2

Let us show accuracy of approximation notion by example referring in Table (1).

With respect to T_1

$$\text{If } X = \{5\} \Rightarrow \underline{R}(X) = \emptyset, \overline{R}(X) = \{2,5\}, \alpha(X) = \left| \frac{\underline{R}(X)}{\overline{R}(X)} \right| = 0,$$

$$X = \{1,3,4\} \Rightarrow \underline{R}(X) = \{4\}, \overline{R}(X) = \emptyset, \alpha(X) = \left| \frac{\underline{R}(X)}{\overline{R}(X)} \right| = 0,$$

$$X = \{2,3,4\} \Rightarrow \underline{R}(X) = \{4\}, \overline{R}(X) = \emptyset, \alpha(X) = \left| \frac{\underline{R}(X)}{\overline{R}(X)} \right| = 0, \&$$

$$\text{if } X = \{1,2,5,6\} \Rightarrow \underline{R}(X) = \{1,2,5,6\}, \overline{R}(X) = \emptyset, \alpha(X) = \left| \frac{\underline{R}(X)}{\overline{R}(X)} \right| = 0$$

With respect to T_2

$$\text{If } X = \{5\} \Rightarrow \underline{R}(X) = \emptyset, \overline{R}(X) = \{2,5\}, \alpha(X) = \left| \frac{\underline{R}(X)}{\overline{R}(X)} \right| = 0,$$

$$X = \{1,3,4\} \Rightarrow \underline{R}(X) = \emptyset, \overline{R}(X) = \emptyset, \alpha(X) = \left| \frac{\underline{R}(X)}{\overline{R}(X)} \right| = 1,$$

$$X = \{2,3,4\} \Rightarrow \underline{R}(X) = \emptyset, \overline{R}(X) = \emptyset, \alpha(X) = \left| \frac{\underline{R}(X)}{\overline{R}(X)} \right| = 1 \&$$

$$\text{if } X = \{1,2,5,6\} \Rightarrow \underline{R}(X) = \{2,5\}, \overline{R}(X) = \emptyset, \left| \frac{\underline{R}(X)}{\overline{R}(X)} \right| = 0.$$

With respect to T_3

$$\text{If } X = \{5\} \Rightarrow \underline{R}(X) = \emptyset, \overline{R}(X) = \{2,5\}, \left| \frac{\underline{R}(X)}{\overline{R}(X)} \right| = 0,$$

$$X = \{1,3,4\} \Rightarrow \underline{R}(X) = \emptyset, \overline{R}(X) = \emptyset, \left| \frac{\underline{R}(X)}{\overline{R}(X)} \right| = 1,$$

$$X = \{2,3,4\} \Rightarrow \underline{R}(X) = \emptyset, \overline{R}(X) = \{2,3\}, \left| \frac{\underline{R}(X)}{\overline{R}(X)} \right| = 0 \&$$

$$\text{if } X = \{1,2,5,6\} \Rightarrow \underline{R}(X) = \emptyset, \overline{R}(X) = \{2,5\}, \left| \frac{\underline{R}(X)}{\overline{R}(X)} \right| = 0.$$

It is clear that the relations R_2 are the best among the differences we created for the selected set X according to the accuracy scale $\alpha_B(X)$, where $\alpha_B(X)=1$, and this means that the set X is crisp(precise) with respect to B , otherwise the set X is rough (imprecise) with respect to B . But for the next set, all differences are perfect because the precision scale gives the best measurement and therefore the set X is crisp with respect to B .

4- Quasi Discrete Topology

In this section, we show that a topological space generated by minimal a construction relation in an information system is a special type of topology called a clopen (quasi discrete) topology.

Definition 4.1

A subset $X \subseteq U$ called a rough set in (U, T_R) , if $X^\circ \neq \overline{X}$. Otherwise X is crisp in (U, T_R) .

Notions 4.1

Let (U, T_R) be a topological space, $X \subseteq U$ in (U, T_R) , then we have

- i- A set $X \subseteq U$ is rough in (U, T_R) if $BN(X) \neq \emptyset$.
- ii- A set $X \subseteq U$ is rough in (U, T_R) if $X^\circ \subset X \subset \overline{X}$.

iii- A set $X \subseteq U$ is rough in (U, T_R) if $X \notin T$.

Definition 4.2

Let (U, T_R) be a general knowledge base, $X \subseteq U$ & $R \in \mathcal{R}$.

- (1) X is totally R – *definable* (exact) set if $X^\circ = X = \overline{X}$.
- (2) X is totally R – *undefinable* If $X^\circ = \Phi$ & $\overline{X} = U$.
- (3) X is internally R -definable set if $X^\circ = X$, $\overline{X} \neq X$.
- (4) X is internally R – *undefinable* set if $X^\circ = \Phi$ & $\overline{X} \neq U$.
- (5) X is externally R – *definable* set if $X^\circ \neq X$, $\overline{X} = X$.
- (6) X is externally R – *undefinable* set if $X^\circ \neq \Phi$ & $\overline{X} = U$.
- (7) X is roughly R – *definable* set if $X^\circ \neq \Phi$ & $\overline{X} \neq U$.
- (8) X is R – *indefinable* (rough) set if $X^\circ \neq X$, $\overline{X} \neq X$.

Proposition 4.3[6]

Let $K = (U, \mathcal{R})$ be a general knowledge base, $X \subseteq U$ and $R \in \mathcal{R}$.

- (1) Set X is R – *definable* (roughly R – *definable*, totally R – *undefinable*) if and only if $\text{so}X^C$.
- (2) Set X is externally (internally) R – *undefinable* if X^C is internally (externally) R – *undefinable*.

Proof

- (1) If X is R – *definable* then $\underline{R}(X) \neq \Phi$ & $\overline{R}(X) \neq U$, we show that
- (2) $\underline{R}(X^C) \neq \Phi$ & $\overline{R}(X^C) \neq U$ since $\underline{R}(X) \neq \Phi \Rightarrow \underline{R}(X) = (\overline{R}(X^C))^C \neq \Phi$
 $\Rightarrow \overline{R}(X^C) \neq U$, and since $\overline{R}(X) \neq U \Rightarrow \overline{R}(X) = (\underline{R}(X^C))^C \neq U \Rightarrow \underline{R}(X^C) \neq \Phi$ i.e.
 X^C is R -*definable*.

Also if X is totally R – *undefinable* then $\underline{R}(X) = \Phi$ & $\overline{R}(X) = U$, we show that

- $\underline{R}(X^C) = \Phi$, & $\overline{R}(X^C) = U$ since $\underline{R}(X) = \Phi \Rightarrow (\overline{R}(X^C))^C = \Phi$
 $\Rightarrow \overline{R}(X^C) = U$. And since $\overline{R}(X) = U \Rightarrow (\underline{R}(X^C))^C = U \Rightarrow \underline{R}(X^C) = \Phi$.
- (2) X is externally R – *undefinable* $\Leftrightarrow X^C$ is internally R – *undefinable* $\Rightarrow$
 $\underline{R}(X) \neq \Phi$ & $\overline{R}(X) = U \Leftrightarrow \underline{R}(X^C) = \Phi$ & $\underline{R}(X^C) \neq U \Rightarrow \underline{R}(X) \neq \Phi \Leftrightarrow (\overline{R}(X^C))^C \neq \Phi$
 $\Leftrightarrow \overline{R}(X^C) \neq U$ & $\overline{R}(X) = U \Leftrightarrow (\underline{R}(X^C))^C = \Phi \Leftrightarrow \underline{R}(X^C) = \Phi$.

Also X is internally R – *undefinable* – $\Leftrightarrow X^C$ is externally R – *undefinable* $\Rightarrow$

$$\underline{R}(X) = \Phi \Leftrightarrow (\overline{R}(X^C))^C = U \Leftrightarrow \overline{R}(X^C) = U \& \overline{R}(X) \neq U \Leftrightarrow (\underline{R}(X^C))^C \neq \Phi \Leftrightarrow \underline{R}(X^C) \neq \Phi.$$

From above example we have

Table summarizing interior, closure, boundary, Exterior, Category, and Accuracy of roughness of sets, W, X, Y, Z in (U, T_R) , such that

$$W = \{2\}, X = \{2,3,4\}, Y = \{1,2\}, Z = \{2,5\}.$$

w.r.t. T_1

Set	<i>Int</i>	<i>Clo</i>	<i>BN</i>	<i>EX</i>	Category	Accuracy
W	$\emptyset$	$\{2,5\}$	$\{2,5\}$	$\{1,3,4,6\}$	Internally <i>R-undefinable</i>	0
X	$\{4\}$	$\{2,3,4,5\}$	$\{2,3,5\}$	$\{1,6\}$	Roughly <i>R-definable</i>	0
Y	$\emptyset$	$\{1,2,5\}$	$\{1,2,5\}$	$\{3,4,6\}$	Internally <i>R-undefinable</i>	1
Z	$\{2,5\}$	$\{2,5\}$	$\emptyset$	$\{1,3,4,6\}$	Roughly <i>R-definable</i>	1

Table (8)

w.r.t. T_2

Set	<i>Int</i>	<i>Clo</i>	<i>BN</i>	<i>EX</i>	Category	Accuracy
W	$\emptyset$	$\{2,5\}$	$\{2,5\}$	$\emptyset$	Internally <i>R-undefinable</i>	0
X	$\emptyset$	U	U	$\emptyset$	Totally <i>R-undefinable</i>	1
Y	$\emptyset$	U	U	$\emptyset$	Totally <i>R-undefinable</i>	1
Z	$\{2,5\}$	$\{2,5\}$	$\emptyset$	$\emptyset$	Roughly <i>R-definable</i>	0

Table (9)

$$T_3 = \left\{ U, \emptyset, \{1\}, \{2\}, \{4\}, \{1,2\}, \{1,3\}, \{1,4\}, \{1,6\}, \{2,3\}, \{2,4\}, \right. \\ \left. \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,2,6\}, \{1,3,4\}, \{1,3,6\}, \{1,4,6\}, \right. \\ \left. \{2,3,4\}, \{1,2,3,4\}, \{1,3,4,6\}, \{1,2,4,6\}, \{3,4,5\} \right\} \\ T_3^C = \left\{ U, \emptyset, \{2,5\}, \{3,5\}, \{5,6\}, \{1,2,6\}, \{1,5,6\}, \{2,3,5\}, \{2,4,5\}, \{2,5,6\}, \right. \\ \left. \{3,4,5\}, \{3,5,6\}, \{4,5,6\}, \{1,2,5,6\}, \{1,3,5,6\}, \{1,4,5,6\}, \{2,3,4,5\}, \right. \\ \left. \{2,3,5,6\}, \{2,4,5,6\}, \{3,4,5,6\}, \{1,2,3,5,6\}, \{1,3,4,5,6\}, \{2,3,4,5,6\} \right\}.$$

w.r.t. T_3

Set	<i>Int</i>	<i>Clo</i>	<i>BN</i>	<i>EX</i>	Category	Accuracy
W	$\emptyset$	$\{2,5\}$	$\{2,5\}$	$\{1,3,4,6\}$	Internally R – <i>undefinable</i>	1
X	$\{2,3,4\}$	$\{2,3,4,5\}$	$\{5\}$	$\{1,6\}$	Roughly R – <i>definable</i>	$\frac{3}{4}$
Y	$\{1,2\}$	$\{1,2,3,5,6\}$	$\{3,5,6\}$	$\{4\}$	Roughly R – <i>definable</i>	$\frac{2}{5}$
Z	$\emptyset$	$\{2,5\}$	$\{2,5\}$	$\{1,3,4,6\}$	Internally R – <i>undefinable</i>	0

Table (10)

Conclusion

We have shown in this paper that the rough set is a standard model in information systems. This model has helped to constructing a topological space by using relationships resulting from the difference and generating the minimum of the construction set and we proved that the topological resulting is more accurate and stronger and is used to study the characteristics of the rough set. where some approximations were compared using the precision scale in order to determine the best.

Reference

- [1] Kelley J.L. (1975), **General topology**, Graduate Texts in Mathematics, ISSN 0072-5285, Editorial universitaria de Buenos Aires.
- [2] Lashin E.F., Kozae A.M., Khadra A. and Medhat A. A. (2005), **Rough set theory for topological spaces**, International Journal of Approximate Reasoning, vol. 40, pp. 25-43.
- [3] Pawlak Z. (1982), **Rough sets**, International Journal of Computer and Information Science, vol. 11, no.,pp. 341-356.

- [4] Pawlak Z. (1982), ***Rough sets***, International Journal of Information And Computer Science, vo.11: no.(5) pp. 341-356.
- [5] Pawlak Z. (1991), ***Rough sets theoretical aspects of reasoning about data***, Vo.9 of System Theory, Knowledge Engineering and Problem Solver Kluwer Academic Publishers, Dordrecht.
- [6] Rashed R. (2013), ***On Some Topological Approximations in Information Systems*** , Mathematics Department, Al-zawia University, Libya.
- [7] Sierpinski W. and Krieger C. (1952), ***General topology***, University of Toronto Press.
- [8] Yao Y. Y., ***Two views of the theory of rough sets in finite universes***, International Journal of Approximate Reasoning, vol.15, no.4, (1996), pp.191-317.
- [9] Yao Y. Y., ***Information granulation and rough set approximation***, International Journal of Intelligent Systems, vol.16, no.1, (2001), pp. 87-104.